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$$\vec{\mathbf{F}}_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{|\vec{\mathbf{r}}_1-\vec{\mathbf{r}}_2|^2}\cdot\frac{\vec{\mathbf{r}}_1-\vec{\mathbf{r}}_2}{|\vec{\mathbf{r}}_1-\vec{\mathbf{r}}_2|}\ [N]$$

$$\boxed{\vec{\mathbf{F}} = \frac{q}{4\pi\varepsilon_0}\int_V \frac{\vec{\mathbf{r}}_1-\vec{\mathbf{r}}_2}{|\vec{\mathbf{r}}_1-\vec{\mathbf{r}}_2|}\rho(\vec{\mathbf{r}}_1)d^3\vec{\mathbf{r}}_1}\ [N]$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\vec{\mathbf{r}}_1-\vec{\mathbf{r}}_2}{|\vec{\mathbf{r}}_1-\vec{\mathbf{r}}_2|^3}\rho(\vec{\mathbf{r}}_1)d^3\vec{\mathbf{r}}_1\ \left[\frac{N}{C}\right]$$

$$\vec{\mathbf{E}} = \lim_{q\to 0}\frac{\vec{\mathbf{F}}}{q}\ \left[\frac{N}{C}\right]$$

$$\int_V \vec{\nabla}\cdot\vec{\mathbf{E}}d^3r = \frac{1}{\varepsilon_0}\int \rho d^3r\ \boxed{\oint_S \vec{\mathbf{E}}\cdot\vec{\mathbf{n}}\cdot ds = \frac{1}{\varepsilon_0}\int \rho d^3r}$$

$$\vec{\mathbf{E}} = -\vec{\nabla}\cdot\phi \qquad \vec{\nabla}\cdot\vec{\mathbf{E}} = \rho/\varepsilon_0$$

$$\vec{\mathbf{E}} = -\vec{\nabla}\cdot\phi \to \vec{\nabla}\cdot\vec{\mathbf{E}} = -\vec{\nabla}^2\phi \to \vec{\nabla}^2\phi = -\frac{\rho}{\varepsilon_0}$$

$$\boxed{\phi(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\vec{\mathbf{r}}_1)}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}_1|}d^3\vec{\mathbf{r}}_1}\ [V]$$

$$W_j = \sum_{i=1}^{j-1}\frac{q_j}{4\pi\varepsilon_0}\frac{q_i}{r_{ij}}\ [J]$$

$$W = \sum_{j=1}^{N} W_j = \sum_{j=1}^{N}\sum_{i=1}^{j-1}\frac{q_j}{4\pi\varepsilon_0}\frac{q_i}{r_{ij}} = \sum_{j=1}^{N}\sum_{i=1}^{j-1} q_i\phi_i$$

$$\boxed{W = \frac{1}{2}\sum_{i\neq j}\frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}}} = \frac{1}{2}\sum_{i\neq j} q_i\phi_j\ [J]$$

$$U = \frac{1}{2}\int \rho\phi d^3r$$

$$U = \frac{1}{2}C\Delta\phi^2 = \frac{1}{2}C\frac{Q^2}{C^2} = \frac{1}{2}\frac{Q^2}{C}\ [J]$$

$$\vec{\mathbf{P}} \equiv \lim_{\Delta V\to 0}\frac{1}{\Delta V}\sum_i \vec{\mathbf{p}}_i$$

$$\rho_P = -\vec{\nabla}\cdot\vec{\mathbf{P}}\ \left[\frac{C}{m^3}\right] \qquad \sigma_P = \vec{\mathbf{P}}\cdot\vec{\mathbf{n}}\ \left[\frac{C}{m^2}\right]$$

$$\phi = \frac{1}{4\pi\varepsilon_0}\oint \frac{\sigma_P(\vec{\mathbf{r}}_1)}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}_1|}dS_1 + \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{\mathbf{r}}_1)}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}_1|}d^3\vec{\mathbf{r}}_1$$

$$\rho_T = \rho + \rho_P \quad \boxed{\oint_S \vec{\mathbf{D}}\cdot\vec{\mathbf{n}}\cdot ds = \int \rho d^3r}$$

$$\vec{\mathbf{D}} = \varepsilon_0\vec{\mathbf{E}} + \vec{\mathbf{P}}\ \left[\frac{C}{m^2}\right] \qquad \vec{\mathbf{P}} = \varepsilon_0\chi_e\vec{\mathbf{E}}$$

$$1+\chi_e = k = \varepsilon_r \qquad \varepsilon_0\varepsilon_r = \varepsilon$$

$$C = \frac{Q}{\Delta\phi} = \frac{\sigma S}{\varepsilon d} = \frac{S\varepsilon}{d}\ [F] = \left[\frac{C}{V}\right]$$

$$\boxed{W_e = \frac{1}{2}\int_V \vec{\mathbf{D}}\cdot\vec{\mathbf{E}}d^3r = \frac{1}{2}\int \rho\phi d^3r = \frac{1}{2}\int E^2\varepsilon d^3r}$$

$$I = \int \vec{\mathbf{J}}\cdot\vec{\mathbf{n}}ds\ [A]$$

$$\vec{\mathbf{J}} = \sum_i q_i N_i \vec{\mathbf{v}}_i \left[\frac{A}{m^2}\right] \to \vec{\mathbf{J}} = g\vec{\mathbf{E}}$$

$$\boxed{\vec{\nabla}\cdot\vec{\mathbf{J}} + \frac{\partial\rho}{\partial t} = 0}$$

Lleis de Kirchhoff:

$$1.\ \vec{\nabla}\cdot\vec{\mathbf{J}} = 0 \to \int \vec{\nabla}\cdot\vec{\mathbf{J}}d^3r = \oint \vec{\mathbf{J}}\cdot\vec{\mathbf{n}}ds = \sum_i^N I_i = 0$$

$$2.\ \int\left(\vec{\nabla}\times\vec{\mathbf{E}}\right)\cdot\vec{\mathbf{n}}ds \oint_C \vec{\mathbf{E}}\cdot d\vec{l} = \sum_i^N V_i = 0$$

$$\vec{\mathbf{F}}_2 = \frac{\mu_0}{4\pi}I_2 I_1\oint_{C1}\oint_{C2}\frac{d\vec{\mathbf{l}}_2\times\left[d\vec{\mathbf{l}}_1\times(\vec{\mathbf{r}}_2-\vec{\mathbf{r}}_1)\right]}{|\vec{\mathbf{r}}_2-\vec{\mathbf{r}}_1|^3}$$

$$\boxed{\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi}\int_V \vec{\mathbf{J}}(\vec{\mathbf{r}})\times\frac{\vec{\mathbf{r}}-\vec{\mathbf{r}}_1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}|^3}d^3r_1}\ [Tesla]$$

$$\vec{\mathbf{F}} = \int \vec{\mathbf{J}}(\vec{\mathbf{r}})\times\vec{\mathbf{B}}(\vec{\mathbf{r}})d^3r\ [N]$$

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}}\times\vec{\mathbf{B}})\ [N]$$

$$\vec{\nabla}\times\vec{\mathbf{B}} = -\frac{\mu_0}{4\pi}\vec{\nabla}\frac{\partial}{\partial t}\int_V \frac{\rho(\vec{\mathbf{r}}_1)}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}_1|}d^3r_1 + \mu_0\vec{\mathbf{J}}(\vec{\mathbf{r}}) =$$

$$= -\varepsilon_0\mu_0\frac{\partial}{\partial t}\vec{\nabla}\phi + \mu_0\vec{\mathbf{J}}(\vec{\mathbf{r}}) = \varepsilon_0\mu_0\frac{\partial\vec{\mathbf{E}}}{\partial t} + \mu_0\vec{\mathbf{J}}(\vec{\mathbf{r}})$$

$$\oint_C \vec{\mathbf{B}}\cdot d\vec{l} = \mu_0\int_S \vec{\mathbf{J}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{n}}ds \to \boxed{\vec{\nabla}\times\vec{\mathbf{B}} = \mu_0\cdot\vec{\mathbf{J}}}$$

$$\to \vec{\nabla}\cdot\vec{\mathbf{J}} = 0$$

$$\boxed{\vec{\nabla}\cdot\vec{\mathbf{B}} = 0} \qquad \boxed{\vec{\nabla}\cdot\vec{\mathbf{D}} = \rho}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi}\int \frac{\vec{\mathbf{J}}(\vec{\mathbf{r}}_1)}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}_1|}d^3r_1 \to \boxed{\vec{\mathbf{B}} = \vec{\nabla}\times\vec{\mathbf{A}}}$$

$$\vec{\mathbf{J}} = 0 \to \vec{\nabla}\times\vec{\mathbf{B}} = 0 \to \vec{\mathbf{B}} = -\mu_0\cdot\vec{\nabla}\phi_m \to \nabla^2\phi_m$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi}\int \frac{(\vec{\mathbf{m}}\times\vec{\mathbf{r}})}{r^3}$$

$$\vec{\mathbf{m}} \equiv \frac{1}{2}I\oint \vec{\mathbf{r}}_1\times d\vec{\mathbf{l}}_1$$

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\left[\frac{3(\vec{\mathbf{m}}\cdot\vec{\mathbf{r}})\vec{\mathbf{r}}}{r^5} - \frac{\vec{\mathbf{m}}}{r^3}\right] = -\mu_0\vec{\nabla}\left(\frac{\vec{\mathbf{m}}\cdot\vec{\mathbf{r}}}{4\pi r^3}\right)$$

$$\vec{\mathbf{m}}_i \equiv \frac{1}{2}\int_{\Delta V}\vec{\mathbf{r}}_1\times\vec{\mathbf{J}}(\vec{\mathbf{r}}_1)d^3r_1$$

$$\boxed{\vec{\mathbf{M}} \equiv \frac{1}{\Delta V}\sum_i \vec{\mathbf{m}}_i = \frac{1}{\Delta V}\frac{1}{2}\int_{\Delta V}\vec{\mathbf{r}}\times\vec{\mathbf{J}}d^3r}\ [H]$$

$$\vec{\mathbf{J}}_M \equiv \vec{\nabla}\times\vec{\mathbf{M}} \qquad \vec{\mathbf{K}}_M \equiv \vec{\mathbf{M}}\times\vec{\mathbf{n}}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi}\int_V \frac{\vec{\mathbf{J}}_m}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}_1|}d^3r_1 + \frac{\mu_0}{4\pi}\int_S \frac{\vec{\mathbf{K}}_m}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}_1|}d^3r_1$$

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$$\vec{\mathbf{B}}(\vec{\mathbf{r}})=\frac{\mu_0}{4\pi}\int_V \rho_M(\vec{\mathbf{r}}_1)\frac{\vec{\mathbf{r}}-\vec{\mathbf{r}}_1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}_1|^3}d^3r_1+\frac{\mu_0}{4\pi}\oint_S \sigma_M(\vec{\mathbf{r}}_1)\frac{\vec{\mathbf{r}}-\vec{\mathbf{r}}_1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}_1|^3}dS_1$$

$$\rho_M=-\vec{\nabla}\cdot\vec{\mathbf{M}}\left[\frac{A}{m^2}\right]\quad \sigma_M=\vec{\mathbf{M}}\cdot\vec{\mathbf{n}}\left[\frac{A}{m}\right]$$

$$\vec{\mathbf{B}}(\vec{\mathbf{r}})=\frac{\mu_0}{4\pi}\int_V \rho_M\frac{\vec{\mathbf{r}}-\vec{\mathbf{r}}_1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}_1|^3}d^3r_1+\frac{\mu_0}{4\pi}\oint_S(\vec{\mathbf{r}}_1)\frac{\vec{\mathbf{r}}-\vec{\mathbf{r}}_1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}_1|^3}dS_1+\mu_0\vec{\mathbf{M}}$$

$$\vec{\mathbf{J}}_T=\vec{\mathbf{J}}_M+\vec{\mathbf{J}}$$

$$\vec{\mathbf{H}}\equiv\frac{1}{\mu_0}\vec{\mathbf{B}}-\vec{\mathbf{M}}\rightarrow\boxed{\vec{\nabla}\times\vec{\mathbf{H}}=\vec{\mathbf{J}}}$$

$$\int\left(\vec{\nabla}\times\vec{\mathbf{H}}\right)\cdot\vec{\mathbf{n}}\cdot ds=\oint\vec{\mathbf{H}}\cdot d\vec{\mathbf{l}}$$

$$\oint\vec{\mathbf{H}}\cdot d\vec{\mathbf{l}}=\int\vec{\mathbf{J}}\cdot\vec{\mathbf{n}}ds$$

$$\boxed{\vec{\mathbf{H}}=\frac{1}{4\pi}\int\rho_M(\vec{\mathbf{r}}_1)\frac{\vec{\mathbf{r}}-\vec{\mathbf{r}}_1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}_1|^3}d^3r_1}\left[\frac{A}{m}\right]$$

$$\vec{\nabla}\times\vec{\mathbf{H}}=0\qquad\vec{\nabla}\cdot\vec{\mathbf{H}}=-\vec{\nabla}\cdot\vec{\mathbf{M}}=\rho_M$$

$$\vec{\mathbf{B}}=\mu_0(\vec{\mathbf{H}}+\vec{\mathbf{M}})\qquad\vec{\mathbf{M}}=\chi_m\vec{\mathbf{H}}$$

$$1+\chi_m=\mu_r\qquad\mu_0\mu_r=\mu\qquad\vec{\mathbf{B}}=\mu\cdot\vec{\mathbf{H}}$$

$$\oint\vec{\mathbf{E}}\cdot d\vec{\mathbf{l}}=\xi=f.e.m.=-\frac{d}{dt}\int_S\vec{\mathbf{B}}\cdot\vec{\mathbf{n}}ds$$

$$\xi=f.e.m.=-\frac{d\Phi}{dt}\rightarrow\Phi=\vec{\mathbf{B}}\cdot\vec{\mathbf{n}}dS\ [Wb]$$

$$\boxed{\vec{\nabla}\times\vec{\mathbf{E}}=-\frac{\partial\vec{\mathbf{B}}}{\partial t}}\quad\oint\left(\vec{\mathbf{E}}-\vec{\mathbf{v}}\times\vec{\mathbf{B}}\right)d\vec{l}=-\int_S\frac{\partial\vec{\mathbf{B}}}{\partial t}\cdot\vec{\mathbf{n}}ds$$

$$\vec{\mathbf{M}}_{21}=\frac{\mu_0}{4\pi_1}\oint_{C2}\oint_{C1}\frac{d\vec{\mathbf{l}}_2\cdot d\vec{\mathbf{l}}_2}{|\vec{\mathbf{r}}_2-\vec{\mathbf{r}}_1|}=L\ [H]$$

$$\Phi_{21}=M_{21}\cdot I_1\quad\Phi_{12}=M_{12}\cdot I_2$$

$$\Phi=LI\ ;\ M_{ij}=\frac{d\Phi_{ij}}{dI}\ [H]$$

$$\boxed{W_m=\frac{1}{2}\int\vec{\mathbf{H}}\cdot\vec{\mathbf{B}}d^3r}$$

Ecuacions de Maxwell en el buit

$$\vec{\nabla}\cdot\vec{\mathbf{E}}=\rho/\varepsilon_0\qquad\vec{\nabla}\times\vec{\mathbf{E}}=-\frac{\partial\vec{\mathbf{B}}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{\mathbf{B}}=0\qquad\vec{\nabla}\times\vec{\mathbf{B}}=\mu_0\vec{\mathbf{J}}+\mu_0\varepsilon_0\frac{\partial\vec{\mathbf{E}}}{\partial t}$$

Ecuacions de Maxwell en medis materials

$$\vec{\nabla}\cdot\vec{\mathbf{D}}=\rho\qquad\vec{\nabla}\times\vec{\mathbf{E}}=-\frac{\partial\vec{\mathbf{B}}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{\mathbf{B}}=0\qquad\nabla\times\vec{\mathbf{H}}=\vec{\mathbf{J}}+\frac{\partial\vec{\mathbf{D}}}{\partial t}$$

$$\vec{\mathbf{F}}=q(\vec{\mathbf{E}}+\vec{\mathbf{v}}\times\vec{\mathbf{B}})\ [N]$$

$$\vec{\mathbf{H}}\equiv\frac{1}{\mu_0}\vec{\mathbf{B}}-\vec{\mathbf{M}}=\frac{\vec{\mathbf{B}}}{\mu}\left[\frac{A}{m}\right]$$

$$\vec{\mathbf{D}}=\varepsilon_0\vec{\mathbf{E}}+\vec{\mathbf{P}}=\varepsilon\vec{\mathbf{E}}\left[\frac{C}{m^2}\right]$$

$$\phi(\vec{\mathbf{r}},t)=\frac{1}{4\pi\varepsilon_0}\int_V\frac{\rho(\vec{\mathbf{r}}',t')}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}\cdot d^3r'\ ;\ t'=t-\frac{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}{c}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t)=\frac{\mu_0}{4\pi}\int_V\frac{\vec{\mathbf{J}}(\vec{\mathbf{r}}',t')}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}\cdot d^3r'\ ;\ t'=t-\frac{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}{c}$$

$$\vec{\mathbf{A}}'=\vec{\mathbf{A}}+\vec{\nabla}\cdot\chi$$

$\vec{\nabla}\cdot\vec{\mathbf{A}}'=0\rightarrow$ Condició de Coulomb

$$\nabla^2\chi=-\vec{\nabla}\cdot\vec{\mathbf{A}}$$

$\vec{\nabla}\cdot\vec{\mathbf{A}}+\nabla^2\chi+\varepsilon\mu\frac{\partial\phi}{\partial t}=\varepsilon\mu\frac{\partial^2\phi}{\partial t^2}\rightarrow$ Condició de Lorentz

Teorema de Poynting

$$\boxed{\int_V\vec{\mathbf{J}}\cdot\vec{\mathbf{E}}d^3r+\frac{1}{2}\frac{\partial}{\partial t}\int(\vec{\mathbf{E}}\cdot\vec{\mathbf{D}}+\vec{\mathbf{B}}\cdot\vec{\mathbf{H}})d^3r=-\oint(\vec{\mathbf{E}}\times\vec{\mathbf{H}})\cdot\vec{\mathbf{n}}ds}$$

$$\boxed{W=\frac{1}{2}(\vec{\mathbf{E}}\cdot\vec{\mathbf{D}}+\vec{\mathbf{B}}\cdot\vec{\mathbf{H}})}$$

$$\boxed{\vec{\mathbf{S}}=\vec{\mathbf{E}}\times\vec{\mathbf{H}}}\rightarrow\langle S\rangle=\frac{1}{T}\int_0^T Sdt$$

$$\nabla^2\vec{\mathbf{E}}-\varepsilon\mu\frac{\partial^2\vec{\mathbf{E}}}{\partial t^2}=0\ ;\ \nabla^2\vec{\mathbf{H}}-\varepsilon\mu\frac{\partial^2\vec{\mathbf{H}}}{\partial t^2}=0$$

$$\vec{\mathbf{E}}=\vec{\mathbf{E}}_0e^{i(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}-\omega t)}\ ;\ \vec{\mathbf{H}}=\vec{\mathbf{H}}_0e^{i(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}-\omega t)}$$

$$\boxed{k^2=\omega^2\varepsilon\mu}$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{E}}=0\qquad\vec{\mathbf{k}}\cdot\vec{\mathbf{H}}=0$$

$$\vec{\mathbf{k}}\times\vec{\mathbf{E}}=\omega\vec{\mathbf{B}}\qquad\vec{\mathbf{k}}\times\vec{\mathbf{H}}=-\omega\vec{\mathbf{D}}$$

$$\boxed{\vec{\mathbf{E}}(\vec{\mathbf{r}},t)=\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\vec{\mathbf{E}}_{\vec{\mathbf{k}}\cdot\omega}e^{i(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}-\omega t)}d^2kd\omega}$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{D}}_{k\cdot\omega}=-i\rho_{k\cdot\omega}\qquad\vec{\mathbf{k}}\cdot\vec{\mathbf{B}}_{k\cdot\omega}=0$$

$$\vec{\mathbf{k}}\times\vec{\mathbf{E}}_{k\cdot\omega}=\omega\vec{\mathbf{B}}_{k\cdot\omega}\qquad\vec{\mathbf{k}}\times\vec{\mathbf{H}}_{k\cdot\omega}=-i\vec{\mathbf{J}}_{k\cdot\omega}-\omega\vec{\mathbf{D}}_{k\cdot\omega}$$

Condicions de frontera:

1.
$$\vec{\mathbf{n}}_{21}\cdot\left(\vec{\mathbf{D}}_1-\vec{\mathbf{D}}_2\right)=\sigma\rightarrow D_{1n}-D_{2n}=\sigma$$
$$\vec{\mathbf{n}}_{21}\times\left(\vec{\mathbf{E}}_1-\vec{\mathbf{E}}_2\right)=0\rightarrow E_{1t}=E_{2t}$$

2.
$$\vec{\mathbf{n}}_{21}\cdot\left(\vec{\mathbf{B}}_1-\vec{\mathbf{B}}_2\right)=0\rightarrow B_{1n}=B_{2n}$$
$$\vec{\mathbf{n}}_{21}\times\left(\vec{\mathbf{H}}_1-\vec{\mathbf{H}}_2\right)=\vec{\mathbf{k}}\rightarrow H_{1t}-H_{2t}=\vec{\mathbf{k}}$$