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$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$; $\vec{\mathbf{L}} = \vec{\mathbf{r}} \wedge \vec{\mathbf{p}}$; $\vec{\mathbf{N}} = \vec{\mathbf{r}} \wedge \vec{\mathbf{F}}$; $V(x) = -\int F dx$;

$$V(x) = \frac{V''(0)}{2}x^2 = \frac{1}{2}kx^2$$

1. OSCIL·LADOR HARMÒNIC SIMPLE

$$\begin{matrix} F = -kx \\ F = m\ddot{x} \end{matrix} \left| \begin{matrix} m\ddot{x} + kx = 0 \\ \ddot{x} + \frac{k}{m}x = 0 \end{matrix} \right| \rightarrow \begin{bmatrix} x_1 = \cos \omega t \\ x_2 = \sin \omega t \\ \omega = (k/m)^{1/2} \end{bmatrix}$$

$$x = A\cos\omega t + B\sin\omega t \rightarrow \boxed{x = x_0 \cos\omega t + \frac{V_0}{\omega}\sin\omega t}$$

2. OSCIL·LADOR HARMÓNIC ESMORTEÏT

$$\begin{matrix} F_f = -\alpha\dot{x} \\ F = -kx \\ F = m\ddot{x} \end{matrix} \rightarrow \left| \begin{matrix} m\ddot{x} + kx + \alpha\dot{x} = 0 \\ \ddot{x} + \frac{\alpha}{m}\dot{x} + \frac{k}{m}x = 0 \end{matrix} \right| \begin{matrix} \gamma = \frac{\alpha}{2m} \\ \omega_0 = (k/m)^{1/2} \end{matrix} \right|$$

$$\boxed{\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0} \quad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2\gamma \pm \sqrt{4\gamma^2 - 4\omega_0^2}}{2}$$

a) OSCIL·LADOR HARMÒNIC SUPRAESMORTEÏT $(\gamma > \omega_0)$

$$\boxed{x(t) = A\exp\left[\left(-\gamma + \sqrt{\gamma^2 - \omega_0^2}\right)t\right] + B\exp\left[\left(-\gamma - \sqrt{\gamma^2 - \omega_0^2}\right)t\right]}$$

b) OSCIL·LADOR HARMÒNIC INFRAESMORTEÏT $(\gamma < \omega_0)$

$$\boxed{x(t) = e^{-\gamma t}\left[A\cos\omega t + B\sin\omega t\right]} \qquad \omega^2 = \omega_0^2 - \gamma^2$$

c) OSCIL·LADOR HARMÒNIC CRÍTICAMENT ESMORTEÏT $(\gamma = \omega_0)$

$$\boxed{x(t) = e^{-\gamma t}[A + Bt]}$$

3. OSCIL·LADOR HARMÒNIC FORÇAT

$$m\ddot{x} + \alpha\dot{x} + kx = F(t) \rightarrow \ddot{x} + \frac{\alpha}{m}\dot{x} + \frac{k}{m}x = \frac{F(t)}{m}$$

$$\boxed{x = e^{-\gamma t}\left[A\cos\omega t + B\sin\omega t\right] + \frac{F_1}{m\delta}\cos(\omega_1 t - \theta_1)}$$

$$\delta^2 = \omega_0^2 - \omega_1^2 + 4\gamma^2\omega_1^2$$

ENERGIA OSCIL·LADOR SIMPLE:

$$\boxed{E = T + V = \tfrac{1}{2}ka^2 = cte}$$

ENERGIA OSCIL·LADOR INFRAESMORTEÏT:

$$\boxed{E(t + T) = E(T)e^{-2\gamma T}}$$

FACTOR QUALITAT: $\boxed{Q = 2\pi\frac{E}{|\Delta E|} = \frac{\omega_1}{2\gamma}}$

POTÈNCIA: $\boxed{\langle P \rangle = \frac{F_1|V_{max}|}{2}\sin\theta_1}$; $\sin\theta_1 = \frac{2\gamma\omega_1}{\delta}$; $\cos\theta_1 = \frac{\omega_0^2 - \omega_1^2}{\delta}$

$\langle P \rangle = m\gamma\omega_1^2 L^2$ on $L = \frac{F_1}{m\delta}$; $\boxed{\langle P \rangle_{\substack{\text{dissipada} \\ \text{per fregament}}} = -\alpha V^2 = m\gamma\omega_1^2 L^2}$

ENERGIA OSCIL·LADOR: $\boxed{E = \tfrac{1}{2}M\omega_1^2 L^2}$

ENERGIA PERDUDA PER PERÍODE:

$$\boxed{\langle P \rangle \tfrac{2\pi}{\omega_1} = 2\pi m\gamma\omega_1 L^2 = |\Delta E|}$$

RESSONÀNCIA: $\omega_0 = \omega_1 \rightarrow \left| \begin{matrix} \delta = 2\gamma\omega_1 \\ \sin\theta_1 = 1 \end{matrix} \right|$ $A = \frac{F_1}{2m\gamma\omega_1} \xrightarrow[\gamma \to 0]{} \infty$

Banda de ressonància: $\Delta\omega = 2\gamma$

FORCES CENTRALS

- **Lleis de conservació:**
  - $E = cte$
  - $L = cte$
  - Moviment pla

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}\ ;\ \vec{\mathbf{r}} = \vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2\ ;\ \vec{\mathbf{R}} = \frac{m_1\vec{\mathbf{r}}_1 + m_2\vec{\mathbf{r}}_2}{m_1 + m_2} \rightarrow \mu\frac{d^2\vec{\mathbf{r}}}{dt^2} = \vec{\mathbf{F}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{n}}$$

EQUACIÓ DEL MOVIMENT:

$$\left| \begin{matrix} m\ddot{r} - mr\dot{\theta}^2 = F(r) \\ mr\ddot{\theta} + 2m\dot{r}\dot{\theta} = 0 \end{matrix} \right| \rightarrow \boxed{\theta = \theta_0 + \int_0^t \frac{L dt}{mr^2}}$$

$$\boxed{F'(r) = F(r) + \frac{L^2}{mr^3}};\ \boxed{V'(r) = V(r) + \frac{L^2}{2mr^2}}$$

EQUACIÓ DE L'ÒRBITA: $\boxed{\frac{d^2U}{d\theta^2} + U = -\frac{m}{L^2U^2}F(U)}$

VELOCITAT AEROLAR:

$\frac{\Delta S}{\Delta t} = cte = \frac{1}{2}rr\frac{d\theta}{dt} = \frac{1}{2}r^2\dot{\theta} = \frac{1}{2}\frac{L}{\mu} = cte = \frac{dS}{dt}$ $\boxed{\frac{S}{T} = \frac{L}{2m}}$ òrbites tancades

PUNTS DE RETORN: $\boxed{\frac{1}{r} = -\frac{mk}{L^2}(1 \pm)}$;

$e =$ excentricitat

1. $k \geq 0, E > 0 \rightarrow \varepsilon > 1 \rightarrow \frac{1}{r} = -\frac{mk}{L^2}(1 - \varepsilon)$ hipèrbola $(-)$ $v > v_0$
2. $k < 0, E > 0 \rightarrow \varepsilon > 1 \rightarrow \frac{1}{r} = -\frac{mk}{L^2}(1 + \varepsilon)$ hipèrbola $(+)$ $v > v_0$
3. $k < 0, E = 0 \rightarrow \varepsilon = 1 \rightarrow \frac{1}{r} = -\frac{2mk}{L^2}(1)$ paràbola $v = v_0$
4. $k < 0, E < 0 \rightarrow \varepsilon < 1 \rightarrow \frac{1}{r} = -\frac{mk}{L^2}(1 \pm \varepsilon)$ el·lipse $v < v_0$

$\varepsilon = \frac{r_{max} - r_{min}}{r_{max} + r_{min}}$; $2a \equiv r_{max} + r_{min}$; $a \equiv$ semieix major; $b \equiv$ semieix menor

HIPÈRBOLA:

$$\left. \begin{matrix} a(\varepsilon^2 - 1) = r(1 + \varepsilon\cos\theta) \\ a = -\frac{k}{2E} > 0 \end{matrix} \right]+ ;\ \left. \begin{matrix} a(\varepsilon^2 - 1) = r(\varepsilon\cos\theta - 1) \\ a = \frac{k}{2E} > 0 \end{matrix} \right]-$$

EL·LIPSE:

$a(1 - \varepsilon^2) = r(1 + \varepsilon\cos\theta)$; $a = \frac{k}{2E} > 0$; $c = a\varepsilon$

PARÀBOLA:

$a = r(1 + \cos\theta)$; $a = \frac{-L^2}{km} > 0$

$$\frac{a^3}{T^2} = \frac{GM_T}{4\pi^2}\ ;\ \frac{L}{2m} = \frac{\pi ab}{T}\ ;\ \tan\left(\frac{\theta}{2}\right) = \left[\frac{mk^2}{2EL^2}\right]^{1/2}\ ;\ v_{escapament} = \sqrt{\frac{2GM}{R}}$$

$$\boxed{dA = 2\pi S dS};\ \boxed{\frac{dN}{N} = n dA};\ \boxed{\left|\frac{dA}{d\theta}\right| = \frac{2\pi(kq_1q_2)^2\sin\theta}{(2mv_0^2)^2\sin^4(\theta/2)}};\ \boxed{\frac{dA}{d\Omega} = \left(\frac{kq_1q_2}{2mv_0^2}\right)\frac{1}{\sin^4\theta/2}}$$

$$\vec{\mathbf{R}} = \frac{1}{M}\sum_i m_i\vec{\mathbf{r}}_i\ ;\ \vec{\mathbf{R}} = \frac{1}{M}\int\vec{\mathbf{r}}dm\ ;\ \vec{\mathbf{p}} = \sum_i\vec{\mathbf{p}}_i = \sum_i m_i\vec{\mathbf{v}}_i = M\vec{\mathbf{V}}_{CM}$$

$$\boxed{\vec{\mathbf{L}} = \vec{\mathbf{R}} \wedge \vec{\mathbf{p}}} \rightarrow \boxed{\vec{\mathbf{L}} = \vec{\mathbf{R}} \wedge \vec{\mathbf{p}} + \vec{\mathbf{L}}_{CM}}$$

$$\boxed{\frac{d^2\vec{\mathbf{r}}}{dt^2} = \frac{d^*\vec{\mathbf{r}}^2}{dt^2} + 2\vec{\boldsymbol{\omega}} \wedge \frac{d^*\vec{\mathbf{r}}}{dt} + \frac{d^*\vec{\boldsymbol{\omega}}}{dt} \wedge \vec{\mathbf{r}} + \vec{\boldsymbol{\omega}} \wedge (\vec{\boldsymbol{\omega}} \wedge \vec{\mathbf{r}})};\ \boxed{\vec{\mathbf{a}}^* = \vec{\mathbf{g}}_e - 2\vec{\boldsymbol{\omega}} \wedge \vec{\mathbf{v}}}$$

$$\boxed{\vec{\mathbf{a}}^* = \frac{d^{*2}\vec{\mathbf{r}}}{dt^2} = \vec{\mathbf{g}} - 2\vec{\boldsymbol{\omega}} \wedge \vec{\mathbf{v}}^* - \vec{\boldsymbol{\omega}} \wedge (\vec{\boldsymbol{\omega}} \wedge \vec{\mathbf{r}})};\ \boxed{\vec{\mathbf{g}}_e = \vec{\mathbf{g}} - \vec{\boldsymbol{\omega}} \wedge (\vec{\boldsymbol{\omega}} \wedge \vec{\mathbf{r}})}$$

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$$\left| \begin{array}{l} M\ddot{R} = \sum F_{ext} \\ \frac{dL_0}{dt} = \sum \vec{\mathbf{N}}_0 = \sum \vec{\mathbf{r}} \wedge F_{ext} \end{array} \right| \boxed{L_z = I_z \dot{\theta}}; \boxed{\frac{dL_z}{dt} = I_z \ddot{\theta}}$$

∈ Moments d'inèrcia additius

∉ STEINER: $\boxed{I = I_G + Md^2}$

∠ EIX PERPEENDICULAR: $I_z + I_x + I_y$

$$\left. \begin{array}{l} T_{translació} = \frac{1}{2} MV_{CM}^2 \\ T_{rotació} = \frac{1}{2} I\vec{\mathbf{\Omega}}^2 \end{array} \right] T_{TOTAL} = T_{translació} + T_{rotació} = \frac{1}{2} MV_{CM}^2 + \frac{1}{2} I\vec{\mathbf{\Omega}}^2$$

TENSOR D'INERCIA:

$$I_{ij} = \iiint \rho dV \begin{bmatrix} (x_2^2 + x_3^2) & -x_1 + x_2 & -x_1 + x_3 \\ -x_1 x_2 & (x_1^2 + x_3^2) & -x_2 x_3 \\ -x_1 x_3 & -x_2 x_3 & (x_2^2 + x_1^2) \end{bmatrix}; \boxed{I\vec{\mathbf{a}} = \lambda \vec{\mathbf{a}}}$$

$$I = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix} \begin{array}{l} (\vec{\mathbf{x}}_1, \vec{\mathbf{x}}_2, \vec{\mathbf{x}}_3): \text{eixos principals d'inèrcia} \\ (I_1, I_2, I_3): \text{moments principals d'inèrcia} \end{array}$$

$$\boxed{I_{ij} = M(\delta_{ij} r^2 - x_i x_j)}; \boxed{T = \frac{1}{2} \iiint_V \rho dV \vec{\mathbf{v}}^2};$$

$$\boxed{L = \iiint \rho dV (\vec{\mathbf{r}} \wedge \vec{\mathbf{v}})} \rightarrow \boxed{\vec{\mathbf{L}} = I\vec{\mathbf{\Omega}}}; \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\boxed{\frac{d\vec{\mathbf{L}}}{dt} = \vec{\mathbf{N}}}; \boxed{\frac{d\vec{\mathbf{L}}}{dt} + \vec{\mathbf{\Omega}} \wedge \vec{\mathbf{L}} = \vec{\mathbf{N}}}; \boxed{\begin{array}{l} I_3 \dot{\Omega}_3 + (I_2 - I_1)\Omega_1 \Omega_2 = N_3 \\ I_2 \dot{\Omega}_2 + (I_1 - I_3)\Omega_1 \Omega_3 = N_2 \\ I_1 \dot{\Omega}_1 + (I_3 - I_2)\Omega_1 \Omega_3 = N_1 \end{array}}$$

$$\boxed{E_{CINÈTICA} + E_{POTENCIAL} = \frac{k}{2} A^2 = cte}; \boxed{\det[a - \omega^2 \mathbf{I}]}$$

$$\frac{\partial^2 U}{\partial t^2} = \frac{T}{\rho} \frac{\partial^2 U}{\partial x^2} \left| \begin{array}{l} T \equiv \text{força; tensió} \\ \rho \equiv \text{densitat lineal massa} \\ v \equiv \text{velocitat} \end{array} \right| \frac{\partial^2 U}{\partial t^2} = v^2 \frac{\partial^2 U}{\partial x^2}$$

$$\boxed{U(x,t) = \left[C \cos\frac{\omega x}{v} + D \sin\frac{\omega x}{v}\right]\left[A \cos\omega t + B \sin\omega t\right]}$$

$$\boxed{U(x,t) = \sin\frac{\omega x}{v}\left[A \cos\omega t + B \sin\omega t\right]}$$

$$\left| \begin{array}{l} A \sin\theta + B \cos\theta = \sin(\theta + \delta) \\ C^2 = A^2 + B^2, \cos\delta = A/C, \sin\delta = B/C \end{array} \right|$$

$$\boxed{\lambda = \frac{2\pi v}{\omega} = \frac{2l}{n}}; \; E = \frac{m}{2}\omega^2 A^2 \rightarrow \rho dx: \boxed{dE = \frac{1}{2} \rho dx \omega^2 A^2}$$

#ones: $\boxed{v = \frac{1}{\lambda}}$; $\boxed{P = \frac{dE}{dt} = \frac{v dE}{dx} = \frac{1}{2} \rho v \omega^2 A^2}$; $\boxed{k = \frac{\omega}{v} = \frac{2\pi}{\lambda}}$;

$$\boxed{\nabla^2 U = \frac{1}{v^2} \frac{\partial^2 U}{\partial t^2}}; \boxed{f = \frac{\omega}{2\pi} = \frac{v}{\lambda} = T^{-1}}; \boxed{T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{\lambda}{v}}$$

$$U(\vec{\mathbf{r}}, t) = f(\vec{\mathbf{U}} \cdot \vec{\mathbf{r}} + vt) + g(\vec{\mathbf{n}} \cdot \vec{\mathbf{r}} - \theta t)$$

$$\boxed{U(\vec{\mathbf{r}}, t) = A \sin(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t + \delta)}$$

$$\boxed{\vec{\mathbf{k}} = k\vec{\mathbf{n}}} \begin{array}{l} \nabla^2 U = -k^2 U \\ \frac{\partial^2 U}{\partial t^2} = -\omega^2 U \end{array}$$